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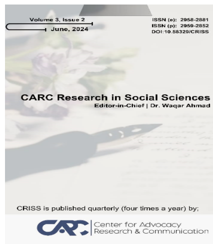

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Comparison and Empirical Evaluation of Classical Tests of Skewness versus Bootstrap Tests of Skewness

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ABSTRACT

This study compares classical (SK-1, SK-2, SSS, MED, STDM, PRSN)and bootstrap (KS and Student's t) tests on the basis of size and power properties for six different data generating processes (chi-square distribution, beta distribution, lognormal distribution, mixture of two normal distribution, and mixture of two uniform and normal distribution) via Monte Carlo Simulations. In general, the classical tests for skewness perform better than bootstrap tests, however, in certain situations the bootstrap tests perform better. Therefore, this study recommends a strategy for choice of test to be applied in different situations. If the data histogram shows deviation from symmetry and the third moment is close to zero then the bootstrap tests should be used. In other cases the classical tests of skewness, in particular SK-2 which is the best performing test, should be used.

1. INTRODUCTION

Skewness of a random variable refers to asymmetry in its probability distribution. Skewness is widely seen as a parallel to the third moment. Whenever the actual value of the third moment is zero, it is assumed that the distribution is symmetric, which means that the measures of central tendency possess an identical value and the basic shape of the distribution on both sides of its center is identical. Nevertheless, symmetry has a broader meaning that cannot be adequately captured by a single point conclusion of facts.

People think that symmetry equates to zero value of third moment, but this is incorrect in practice. Additionally, there are several scenarios in which the distribution on both edges of the center of data varies while the third central moment is zero. Statisticians recognized this problem and developed alternate measures of asymmetry, such as medcouple and split sample skewness. In the literature, skewness tests are divided into two distinct groups: bootstrap and classical testing. Numerous comparison studies using the Monte Carlo simulation approach are being conducted in the past to investigate the size and power aspects of skewness tests.

Adil (2011) assessed five skewness tests based on test power employing the Monte Carlo simulation the strategy. He employed the chi-square, beta, and lognormal distributions for DGPs, with varied skewness options while maintaining the test size fixed. He created a novel skewness test called Split Sample Skewness (SSS) and shown that it was the most effective in identifying skewness. Doane and Seward (2011) evaluated modified standardized and modified Pearsonian skewness tests based on test power, employing the chi-square distribution for DGP using degrees of freedom (df=5) for moderately skewed chi-square and df=2 for highly skewed chi-square. They determined that modified standardized tests are most effective in identifying skewness.

Tabor (2010) examined eleven tests of skewness using

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the normal distribution for the DGP, with an average of 5 and a standard deviation of 1. He used df = 40, df = 5, and df = 1 to represent slightly, moderately, and highly skewed distributions, accordingly. The author discovered that SK-1 and SK-2 outperformed the remaining tests. Brys et al. (2003) examined six skewness tests on the foundation of power, employing the Gaussian distribution for the DGP, and determined that the medcouple test outperformed each of the five tests. Brys et al. Struyf (2004) examined three robust skewness tests employing the Gaussian distribution for the DGP based on test power utilizing the simulation approach and discovered that the medcouple performed well.

Furthermore, Zheng and Gastwirth (2010) evaluated bootstrap tests of skewness using beta distribution with specifications of parameters (2, 2), uniform distribution having specifications of parameters [0, 1], and t-distribution having df=3 for DGPs. They identified that the two-sample t-test performed better. In a similar manner Riaz et al. (2018), Hussan et al. (2019), Waheed et al. (2021), and Akbar et al. (2019) make similar attempts. All of these research investigations mainly compared conventional or bootstrap assessments of skewness. Yet, there is no comparison between the bootstrap and classical tests for skewness. This study aims to address this gap by analyzing six conventional tests and two tests using the bootstrap approach. Also, our study provides a general strategy in which situations classical and bootstrap tests are useful to be applied. Section 2 and 3 describe the classical and bootstrap tests considered while section 4 shows methodology adopted in our study. Section 5 explains simulations results corresponding to different DGP's. Section 6 clarifies the situations through empirical example in which bootstrap tests are useful to apply while section 7 concludes all results derived from our study.

Classical Skewness Tests

Almost all classical skewness of measurement tests are constructed from the central tendency measurements (i.e. mean, median and mode). In this study we have considered six tests from classical skewness category. These tests with their mathematical structure are explained below.

Pearsonian (PRSN) Test

Pearsonian coefficient of skewness (Hereafter, abbreviated PSRN) is introduced by Karl Pearson which takes mean and mode to measure skewness. However, sometime mode is dxifficult to obtain then Stuart and Ord (1994) used median instead of mode to define the test-statistic as,

$$PRSN = \frac{3(mean - \mathrm{m}edian)}{s.d}$$

The value of this test usually varies between -3 and +3. If PRSN=0 then distribution is diagnostic as symmetric. However, if this test statistic value is greater than zero the distribution is identified to be positively skewed, if the test value is less than 0 then distribution is called negatively skewed.

Standardized Moment (STDM) Test

The standardized moment test of skewness (Hereafter, abbreviated STDM) is defined as $\gamma = \mu_3 / \sigma^3$, where μ_3 is third central moment and σ is standard deviation, having its value in between -2 and +2 for most of the distributions. This test was developed by Jarque and Bera (1980). If the value of this test statistic is greater than zero then the distribution is detected as positively skewed. If the test value is less than zero then the distribution` is said to be negatively skewed and for symmetrical distributions the test value is zero.

Medcouple (MED) Test

This test was developed by Brys et al. (2004) which is based on two parts of the data set. Let there is a series $x_1, x_2, x_3, \ldots\ldots., x_n$ which is sorted such that $x_1 \le x_2 \le x_3 \le \ldots\ldots \le x_n$ and let m_n is the median of this series. Then medcouple can be measured as,

$$MC(x_1, x_2, x_3, \ldots\ldots., x_n) = \underset{x_i \le m_n \le x_j}{med}\left(h(x_i, x_j)\right) \text{ ----- (1.1)}$$

Where x_j are the values which are greater than median and x_i are the values which are lesser than median. For all $x_i \ne x_j$ the kernel function h is given by,

$$h(x_i, x_j) = \frac{(x_j - m_n) - (m_n - x_i)}{x_j - x_i} \text{ ------ (1.2)}$$

MC always lies between -1 and 1. Equation (1.2) calculates the (standardized) difference between the distances of x_j and x_i to the median. It is positive if x_j lies further from the median than x_i and $h(x_i, x_j)$ becomes negative if reverse situation occurs. A zero value is attained at the symmetric case where $x_j - m_n = m_n - x_i$. This test hereafter is abbreviated as MED.

Split Sample Skewness (SSS) Test

Adil (2011) presented this robust skewness approach (hence abbreviated SSS), essentially divides the entire set of data into two equal portions and then calculates the interquartile range for each portion by identifying the first and third quartiles on each side (to both sides of the median).

Mathematically it is defined as,

$SSS = \ln(IQR_R / IQR_L)$

Where

$IQR_L = Q_{L3} - Q_{L1}$ and $IQR_R = Q_{R3} - Q_{R1}$

IQR_L and IQR_R represents the interquartile range at the left and right part of the median. Q_{R3}, Q_{R1}, Q_{L3}, and Q_{L3} correspond to the 87.5th, 62.5th, 37.5th, and 12.5th percentiles, respectively. Whenever the result is 0, the pattern of distribution is symmetric; alternatively, it could be negatively or positively skewed.

2.5. Skewness-1 (SK-1) Test

SK-1 test is taken from the study of Tabor (2010), in which SK-1 test with its high power is recognized as best performing test among eleven tests, it is defined as:

$$SK1 = \frac{\max - median}{median - \min}$$

Skewness-2 (SK-2) Test

Similarly, SK-2 is also identified as best test from Tabor (2010)'s results. It is defined as,

$$SK2 = \frac{\frac{1}{2}(\min + \max)}{median}$$

Bootstrap Tests

Bootstrap method for measurement of skewness was introduced by Efron (1979, 1982), further discussed in Efron and Tibshirani (1993). Suppose that $X_i = X_1, X_2, X_3, \ldots\ldots\ldots\ldots, X_n$ and its median is ν. Now, two sub samples are obtained as,

$$X_i^{'} = X_i - \nu \quad \forall\ X_i > \nu, \qquad X_i^{''} = \nu - X_i \quad \forall\ \nu > X_i$$

Under the null hypothesis of same distribution from both sub-samples, X_i is identified to be symmetric if the distribution of $X_i^{'}$ is similar to $X_i^{''}$. The discrepancy between distribution of $X_i^{'}$ and $X_i^{''}$ could be calculated by any two suitable tests. The two tests used in this study are Students's t-test (Hereafter, abbreviated t-test) and Kolmogorov-Smirnov test (Hereafter, abbreviated KS). Modarres (2002) uses following bootstrap approach.

Let $X_i = X_1, X_2, X_3, \ldots\ldots\ldots\ldots, X_n$ is the given series with median ν, then

$$X_i^{'} = X_i - \nu \quad \forall\ X_i > \nu, \quad X_i^{''} = \nu - X_i \quad \forall\ \nu > X_i, \quad Z_i = 2\nu - X_i, \ Y_i = (X_i, Z_i)$$

Y is a vector of size $2n$. A random sample with replacement of size n is generated from the above data $Y_i = (X_i, Z_i)$, two sample t-test (i.e. Student's t-test) and KS test (i.e. Kolmogorov-Smirnov) are applied using the procedure prescribed above and their values are noted. This process is repeated a large number of times (i.e. 10000) times and each time two sample t-test and KS test are applied and the values are noted. And, from these 10000 values 5% critical values of each two-sample t-test and KS test have been calculated. The detail of each test of bootstrap test is given below.

Student's t-Test

This test is introduced by William Sealy Gosset in 1908 which is based on two sub samples. Let $X_i = X_1, X_2, X_3, \ldots\ldots\ldots\ldots, X_n$ is given series with median ν. Now, two sub samples are obtained as,

$$X_i^{'} = X_i - \nu \quad \forall\ X_i > \nu, \qquad X_i^{''} = \nu - X_i \quad \forall\ \nu > X_i$$

And the desired t-test is defined as,

$$t = \frac{(\bar{X}^{'} - \bar{X}^{''})}{s_p\sqrt{\frac{2}{n} + \frac{2}{n}}}$$

Where $\bar{X}^{'}$ and $\bar{X}^{''}$ are the two samples of size $\frac{n}{2}$ and $\frac{n}{2}$ while s_p^2 is the unbiased pooled estimate common variance. Where $s_p^2 = \frac{\left(\frac{n}{2}-1\right)s_1^2 + \left(\frac{n}{2}-1\right)s_2^2}{\frac{n}{2}+\frac{n}{2}-2}$ with $s_1^2 = \frac{1}{(n/2-1)}\sum(X^{'} - \bar{X}^{'})^2$

and $s_2^2 = \frac{1}{(n/2-1)}\sum(X^{''} - \bar{X}^{''})^2$ is the unbiased variance.

Kolmogorov-Smirnov (KS) Test

Kolmogorov-Smirnov (KS) test is based on two sub samples and it is also called two sample test developed by Smirnov (1947). Let $X_i = X_1, X_2, X_3, \ldots\ldots\ldots\ldots, X_n$ is given series having median ν of this sample. Now, two sub samples are obtained as,

$$X_i^{'} = X_i - \nu \ \forall\ X_i > \nu, \qquad X_i^{''} = \nu - X_i \ \forall\ \nu > X_i$$

Then $X_i^{'}$ and $X_i^{''}$ series are arranged in ascending order to obtain $\max\{Y_i\}$ test statistic, where $Y = \left|X_i^{'} - X_i^{''}\right|$.

2. METHODOLOGY

In order to carry out comparison of classical and bootstrap skewness tests, six data generating processes (DGP) have been taken. DGP-1, under the null hypothesis of symmetry (i.e. no skewness), is used to calculate the size of all tests using simulated critical value. The size calculation of all tests under the null hypothesis of symmetry are obtained from three probability distribution; chi-Square distribution with df = 10000, beta distribution with (35, 35) and lognormal distribution with (10, 0.0001).

In order to compare the power of all tests at different alternatives of skewness, five DGP's are taken which are drawn from chi-square distribution, beta distribution, lognormal distribution, mixture of two normal distribution, and mixture of two uniform and normal distribution. First, using DGP-2, a random sample of size n is taken from chi-square distribution with different degrees of freedom υ as this distribution provides good plat form for comparison of tests of skewness. The skewness of chi-square distribution is measured from the third moment that is $\chi^2 = \sqrt{8/\upsilon}$. By putting different values of υ in this expression, different values of third moment theoretical skewness are obtained (see Table 1). Second, DGP-3 is taken from beta distribution with measurement of skewness define as beta= $(2(\beta-\alpha)\sqrt{\alpha+\beta+1})\,/\,((\alpha+\beta+2)\sqrt{\alpha\beta})$ corresponding to different values of α and β (see Table 1) to obtain different theoretical values of skewness. Third, DGP-4 is used from lognormal distribution with measurement of skewness logn $=(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$ corresponding to theoretical values (see Table 1) of parameters of interest (i.e. mean μ and standard deviation σ) to define different level of skewness. Fourth, a mixture of two normal distributions has been taken to define DGP-5 for variable X,

$$Z_i = \begin{Bmatrix} X_{1i} \sim N(\mu_1, \sigma_1^2)\ \textit{with probability}\ \alpha \\ X_{2i} \sim N(\mu_2, \sigma_2^2)\ \textit{with probability}\ (1-\alpha) \end{Bmatrix}$$

A measurement of skewness based on second and third moments for mixture of two normal distributions is defined as $S_{kw} = U_3/(U_2)^{3/2}$ to detect different level of skewness corresponding to different theoretical values of $\alpha, \mu_1, \mu_2, \sigma_1^2$ and σ_2^2 (see Table 1). Finally, DGP-6 is defined from mixture of two uniform distributions and one normal distribution, that is,

$$Z_i = \begin{Bmatrix} X_{1i} \sim U(a,b) \text{ with probability } \alpha \\ X_{2i} \sim N(\mu,\sigma) \text{ with probability } (1-\alpha) \\ X_{3i} \sim U(a_1,b_1) \text{ with probability } (1-\alpha-\beta) \end{Bmatrix}$$

A skewed distribution for the mixture of two uniforms and one normal distributions has obtained by taking different values of $\alpha, \beta, a, b, \mu, \sigma^2, a_1$ and b_1 (see Table 1) by using the measurement of skewness $S_{kw} = U_3/(U_2)^{3/2}$, but it is observed that all tests detect zero skewness.

Table 1

Theoretical Skewness of Distribution at Different Alternatives

Chi-Square Distribution												
skewness	0.25	0.5	0.75	1	1.25	1.5	1.75	2	3			
υ	128	31	14	8	5	4	3	2	1			
Beta Distribution												
Skewness	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
α	14.5	7.5	4.4	2.8	2	1.44	1.1	0.85	0.69	0.56	0.47	0.39
β	35	35	35	35	35	35	35	35	35	35	35	35
Lognormal Distribution												
skewness	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
μ	10	10	10	10	10	10	10	10	10	10	10	10
σ	0.007	0.03	0.06	0.1	0.15	0.2	0.25	0.31	0.36	0.41	0.46	0.52
Mixture of two Normal Distributions												
Skewness	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
α	0.1	0.1	0.1	0.1	0.1	0.12	0.1	0.1	0.8	0.8	0.1	0.1
μ_1	2	2	1.6	2	2	2	6	6.7	0	0	3	3
μ_2	0	0	0	0	0	0	0	0	2	2	0	0
σ_1^2	0.8	0.7	0.7	0.2	0.6	0.8	0.8	0.8	0.2	0.1	0.5	0.9
σ_2^2	1	0.8	0.6	0.5	0.5	0.5	1	0.9	1.2	1.5	0.1	0.1
Mixture of two Uniforms and Normal Distribution												
	skewness	α	β	a	B	μ	σ^2	a_1	b_1			
SK-2	0	0.4	0.4	-5	2	0	0.3	2	4			
STDM	0	0.1	0.4	-11	5	1	0.1	-1	16			
SK-1	0	0.4	0.4	-5	2	1	0.1	1	2			
PRSN	0	0.6	0.9	0	5	1	0.5	-8	0			
SSS	0	0.1	0.9	-1	0	1	0.1	1	6			
MED	0	0.1	0.5	-8	0	0	0.1	-1	2			

Monte Carlo Simulation Design

The Monte Carlo simulation design of this study consists of the following steps.

Calculation of Critical Values for Classical Tests of Skewness

- A symmetric series is generated for respective distributions and all classical tests of skewness are calculated.
- Above step is repeated 10000 times and critical values with 5% level of significance are obtained.

Calculation of Critical Values for Bootstrap Test of Skewness

- Let X_i be the data with median ν.
- Calculate $X' = X_i - \nu \quad if \quad X_i > \nu$ and $X'' = \nu - X_i \quad if \quad \nu > X_i$
- Calculate t-test and KS test using approach mentioned in section 3.
- The series $Z_i = 2\nu - X_i$ is formulated such that Z_i has mirrored distribution of original sample X_i.
- A symmetric series $Y_i = (X_i, Z_i)$ is also formulated and then Y_i has a perfectly symmetric distribution.
- A random sample of size n with replacement is generated.
- Bootstrap tests based on two-sample t-test and KS test using the above prescribed procedure are applied and the values are noted.

- The above process is repeated 10000 times and the values are noted.
- From the above noted values 5% critical values of bootstrap tests based on two-sample t-test and KS test are calculated.

Calculation of Power of Test

- A number of skewed series is generated and all classical and bootstrap tests of skewness are calculated.
- The values of all tests of skewness are compared with simulated critical values obtained in sections 4.1.1 and 4.1.2.
- To find the power of a test, the above steps are simulated 10000 times.

3. RESULTS & DISCUSSION

To provide base for the size of the test for all distributions considered in our study a symmetry is obtained. For this purpose, a high degrees of freedom for chi-square distribution while the same values of parameters of interest for beta distribution are taken to have symmetry distribution and provide a base for the size of test. A similar strategy has been followed for other skewed distributions.

Power Computation in Chi-Square Distribution

At small sample size (i.e. n=40) among classical tests of skewness, SK-2 performs best and attains highest power at all alternatives (see Table 2). Similarly at this degree of freedom among bootstrap tests of skewness two sample t-test performs better. But, when the level of skewness is high, i.e. at degree of freedom 1, then all classical tests have almost same power except MED test. While two sample t-test under the frame work of bootstrap tests also attains high power. But at high level of skewness, i.e. at degree of freedom 1, all classical tests and bootstrap tests attain same power.

A similar situations are observed for both type of skewness tests if sample size is medium (i.e. n=80), in which SK-2 test with high power at each alternate corresponding to different degrees of freedom and skewness is observed as best test. Similarly, KS test with less power gain is identified as bad performer test in both categories. At large sample size (i.e. n=150) among classical tests SK-2, SK-1 and STDM tests have same power when the degree of freedom is 8. At this stage the power of MED test is very poor among classical tests of skewness. While two sample t-test in the bootstrap class tests achieves highest power at all alternatives. However, when the level of skewness becomes higher, i.e. at degree of freedom 1, classical and bootstrap tests attain almost same power. These results are also explained in Figure 1.

Table 2

Power of Tests of Skewness for Chi-Square Distribution

	DF	3rd Moment Theoretical Skewness	Classical Tests						Bootstrap Tests	
			PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Small Sample Size (n=40)	10000	0	5.1	5.5	5.1	5.1	4.9	5.0	5.9	3.0
	128	0.25	8.7	12.5	64.1	12.7	8.1	6.9	6.6	3.8
	31	0.5	16.6	29.2	85.1	29.4	13.7	11.3	10.4	4.3
	14	0.75	26.1	49.2	94.6	47.6	22.7	15.7	17.4	6.7
	8	1	38.8	70.4	98.3	66.3	33.0	22.4	26.9	11.2
	5	1.25	54.1	88.2	99.6	82.4	47.9	30.8	33.0	12.4
	4	1.5	63.0	92.9	99.9	87.2	56.9	37.4	43.6	21.2
	3	1.75	73.8	97.2	100.0	93.1	69.7	47.1	56.1	29.8
	2	2	88.0	99.6	100.0	97.9	86.1	62.4	72.0	44.1
	1	3	98.8	100.0	100.0	99.9	99.2	88.6	92.1	71.9
Medium Sample Size (n=80)	128	0.25	10.7	17.6	71.3	18.9	10.2	7.9	9.6	4.6
	31	0.5	23.9	43.6	92.8	47.8	20.0	14.4	15.3	5.3
	14	0.75	41.2	73.2	99.0	75.6	35.2	22.5	30.4	10.8
	8	1	60.3	92.1	99.9	91.7	53.2	34.3	47.3	21.0
	5	1.25	79.6	99.2	100.0	98.3	72.5	46.5	65.4	36.0
	4	1.5	86.7	99.7	100.0	99.4	81.4	56.2	71.8	43.4
	3	1.75	94.0	100.0	100.0	99.9	91.2	67.5	85.3	62.0
	2	2	98.9	100.0	100.0	100.0	98.5	84.2	95.4	78.5
	1	3	100.0	100.0	100.0	100.0	100.0	98.7	99.8	96.1
Large Sample Size (n=150)	128	0.25	14.0	20.0	77.7	26.0	13.0	10.0	12.4	4.8
	31	0.5	34.7	57.3	97.7	70.1	29.4	18.9	28.0	9.4
	14	0.75	60.9	89.2	99.9	94.0	53.4	31.8	49.0	20.4
	8	1	82.9	99.1	100.0	99.6	75.5	48.6	74.1	41.8
	5	1.25	95.1	100.0	100.0	100.0	91.9	67.4	90.6	68.4
	4	1.5	97.9	100.0	100.0	100.0	96.4	76.9	96.4	81.2
	3	1.75	99.6	100.0	100.0	100.0	99.2	87.6	98.0	89.8
	2	2	100.0	100.0	100.0	100.0	100.0	96.3	99.8	97.3
	1	3	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.7

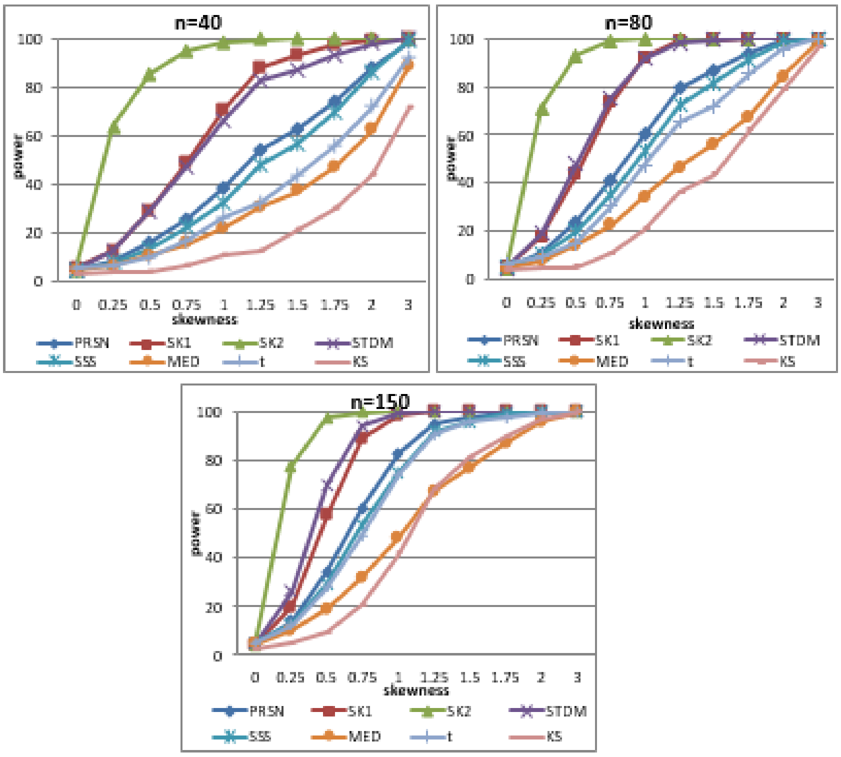

Fig. 1. Power Curve of Classical Tests and Bootstrap Tests in Chi-Square Distribution

At small sample size (n=40) among the classical and bootstrap tests of skewness SK-2 test attains high power at different levels of skewness while KS test performs very poor having less power (see Figure 1). However, as the skewness increases KS test also achieves good power pattern. The power of SK-1 and STDM tests is almost same when the level of skewness is low but when level of skewness increases then SK-1 test performance gets better by attaining high power. When sample size is small then SK-2 test performs better in the category of classical tests of skewness while two sample t-test identifies as best in the bootstrap tests category. While, SK-2 test with high power is detected as best performer test among classical and bootstrap tests at n=40. When sample size becomes medium (i.e. n=80) then SK-2 test attains high power even at low level of skewness. At all level of skewness the power of SK-1 and STDM test is same by attaining same power. The power of KS test is worst by attaining low power at different alternatives of skewness. Among bootstrap tests, the two sample t-test has maximum power at different alternatives of skewness. Again, SK-2 test performs better in both categories by attaining high power at medium sample size.

At large sample size (i.e. n=150) all classical tests of skewness have good power except MED test at different alternatives of skewness. The SSS test and t-test have almost same power by attaining same power at different alternatives of skewness. The KS test attains low power at low level of skewness but its power increases as level of skewness increases and gets higher than MED test as compared to its performance for small and medium samples. Hence, it is concluded that among all tests of skewness the SK-2 test shows better power, while KS test shows poor power for all sample sizes when the level of skewness is low. Also, SSS test did not show satisfactory power at small and medium sample size when the level of skewness is low.

Power Computation in Beta Distribution

At small sample size (n=40), SK-2 test performs best by having high power as compared to other tests when the values of parameters of beta distribution are high (see Table 3). Similarly, in the class of bootstrap tests t-test performs better than KS test in small sample size. But, when the level of skewness is high then all classical tests have almost same power except MED test. At medium sample size (n=80) among classical tests of skewness, SK-2, SK-1 and STDM tests attain high power at low level of skewness. Similarly, t-test performs better at low level of skewness. However, at high level of skewness all classical tests and bootstrap tests attain same power. At large sample size (n=150), among classical tests SK-2, SK-1 and STDM tests have high power when beta distribution has parameters (2.8, 35). At this stage the power of MED test is very poor, while t-test shows high power. But, when the level of skewness becomes higher classical tests and bootstrap test attain almost same power. Moreover, SK-2 test performs best by attaining high power among all tests of skewness when the sample size is small at all alternatives of skewness (see Figure 2). PRSN and SSS tests have almost same power at high level of skewness. KS test performs worst by attaining low power at low level of skewness, but when the levels of skewness becomes high then its power gets better. Among bootstrap tests, t-test performs better by attaining high power than its rival test.

When sample size is 80 then SK -2 test performs best by attaining high power among all tests of skewness while KS test is identified as worst test (see Figure 2). SK-1 and STDM tests have same power at all level of skewness while MED test with less power at all alternatives is identified as worst test in the class of classical test of skewness. In the category of bootstrap tests, t-test with high power at each alternative than KS test is detected as best test. At large sample size (n=150), all classical tests of skewness have good power except MED (classical test) and KS (bootstrap test) tests showing poor performance. Overall, KS test with less power at each alternative is identified as worst performer test in both categories.

Table 3

Power of Tests of Skewness for Beta Distribution

	Parameters	3rd Moment Theoretical Skewness	Classical Tests						Bootstrap Tests	
			PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Small Sample Size (n=40)	(35,35)	0	4.6	5.5	5.7	5.7	4.9	5.0	5.2	2.7
	(14.5,35)	0.25	10.0	14.9	37.7	15.4	8.8	8.0	6.4	3.3
	(7.5,35)	0.5	18.0	33.8	70.8	34.6	16.3	12.3	11.5	4.6
	(4.4,35)	0.75	29.9	59.6	90.8	57.4	26.1	18.2	19.2	7.6
	(2.8,35)	1	43.8	79.6	97.5	75.3	39.3	25.9	28.7	12.2
	(2,35)	1.25	58.2	92.6	99.3	88.1	53.7	35.0	37.9	17.0
	(1.44,35)	1.5	72.0	97.6	99.9	94.3	69.4	46.4	52.2	28.4
	(1.1,35)	1.75	83.7	99.2	100.0	97.6	82.2	58.0	63.3	39.5
	(0.85,35)	2	91.5	99.9	100.0	99.3	90.7	68.8	76.2	50.5
	(0.69,35)	2.25	96.0	100.0	100.0	99.7	96.2	77.5	82.9	60.1
	(0.56,35)	2.5	98.1	100.0	100.0	99.9	98.5	85.1	89.7	70.1
	(0.47,35)	2.75	99.0	100.0	100.0	100.0	99.5	89.9	93.6	75.5
	(0.39,35)	3	99.8	100.0	100.0	100.0	99.9	94.2	95.6	81.5
Medium Sample Size (n=80)	(14.5,35)	0.25	12.0	21.8	46.7	24.4	11.7	9.0	9.0	4.3
	(7.5,35)	0.5	25.5	51.2	85.1	55.6	23.7	15.3	17.4	6.7
	(4.4,35)	0.75	45.9	82.8	98.0	84.0	40.9	25.5	32.2	13.5
	(2.8,35)	1	68.0	96.7	99.9	95.9	62.4	39.9	52.6	22.2
	(2,35)	1.25	82.8	99.5	100.0	99.1	78.3	52.0	68.0	39.1
	(1.44,35)	1.5	93.4	100.0	100.0	99.9	91.7	68.4	82.6	56.6
	(1.1,35)	1.75	97.8	100.0	100.0	100.0	96.8	79.3	89.2	71.0
	(0.85,35)	2	99.4	100.0	100.0	100.0	99.4	88.6	97.3	81.7
	(0.69,35)	2.25	99.9	100.0	100.0	100.0	99.9	94.0	99.0	89.0
	(0.56,35)	2.5	100.0	100.0	100.0	100.0	100.0	97.1	99.8	93.2
	(0.47,35)	2.75	100.0	100.0	100.0	100.0	100.0	98.7	99.9	94.2
	(0.39,35)	3	100.0	100.0	100.0	100.0	100.0	99.6	99.9	97.4
Large Sample Size (n=150)	(14.5,35)	0.25	16.9	27.2	55.0	34.3	14.3	11.2	12.6	5.5
	(7.5,35)	0.5	40.8	70.3	94.5	80.0	34.8	22.1	29.2	11.1
	(4.4,35)	0.75	69.0	96.5	99.9	97.9	60.5	38.7	54.0	24.2
	(2.8,35)	1	89.3	99.9	100.0	99.9	83.6	58.3	79.8	51.2
	(2,35)	1.25	97.4	100.0	100.0	100.0	94.6	74.7	90.8	74.2
	(1.44,35)	1.5	99.6	100.0	100.0	100.0	99.0	87.4	98.7	90.8
	(1.1,35)	1.75	99.9	100.0	100.0	100.0	99.9	95.0	99.7	96.0
	(0.85,35)	2	100.0	100.0	100.0	100.0	100.0	98.4	99.9	98.3
	(0.69,35)	2.25	100.0	100.0	100.0	100.0	100.0	99.6	100.0	98.5
	(0.56,35)	2.5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9
	(0.47,35)	2.75	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9
	(0.39,35)	3	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9

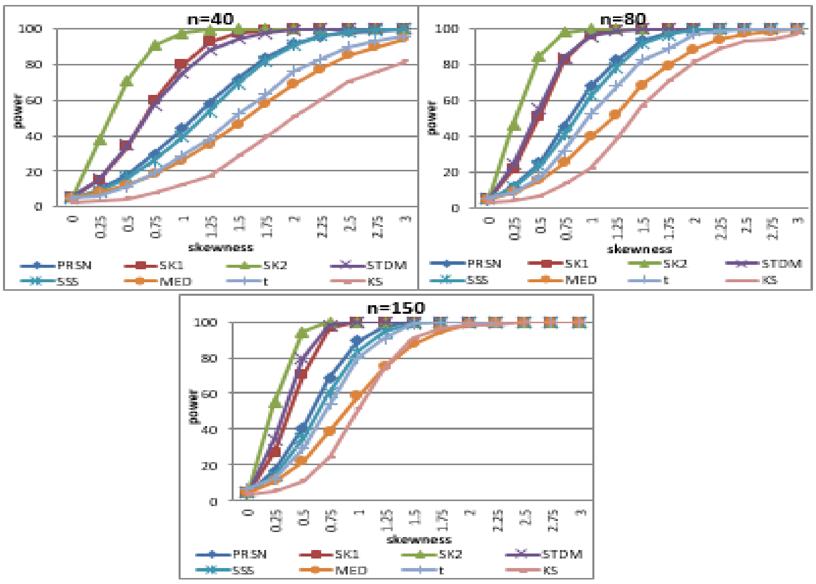

Fig. 2. Power Curve of Classical Tests and Bootstrap Tests in Beta Distribution

Hence, it is observed that SK-2 and KS tests are identified as best and worst performer tests whether sample size is small medium or large when beta distribution is taken into account.

Power Computation in Lognormal Distribution

When lognormal distribution takes into to observe the power performance of classical and bootstrap skewness tests then it is analyzed that SK-2 performs best by attaining high power while SK test is classified as worst performer test (see Table 4).

Table 4

Power of Test of Skewness for Lognormal Distribution

	Parameters	3rd Moment Theoretical Skewness	Classical Tests						Bootstrap Tests	
			PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Small Sample Size (n=40)	(10,.0001)	0	4.7	5.2	5.2	5.1	5.1	4.2	5.7	3.2
	(10,0.007)	0.25	5.3	5.6	50.9	5.5	5.2	4.9	7.4	4.3
	(10,0.03)	0.5	6.1	7.3	57.2	7.6	6.7	5.3	6.5	5.5
	(10,0.06)	0.75	7.3	10.1	64.6	10.6	7.4	5.8	6.9	6.2
	(10,0.1)	1	10.4	16.1	73.9	16.8	9.6	7.8	7.8	7.5
	(10,0.15)	1.25	13.7	26.0	82.9	27.1	13.2	9.1	9.9	8.3
	(10,0.20)	1.5	19.0	36.4	89.4	37.8	18.6	12.0	10.4	8.3
	(10,0.25)	1.75	25.5	49.0	94.0	49.3	23.3	15.0	16.0	9.1
	(10,0.31)	2	33.0	63.3	97.4	63.0	30.9	18.4	19.5	10.8
	(10,0.36)	2.25	40.2	72.8	98.5	71.8	36.5	21.6	25.7	11.6
	(10,0.41)	2.5	48.4	81.3	99.4	80.0	44.3	26.4	30.5	12.1
	(10,0.46)	2.75	55.3	87.9	99.5	85.8	52.1	30.1	37.5	16.3
	(10,0.52)	3	63.5	92.8	99.9	90.8	59.8	35.8	43.8	20.8
Medium Sample Size (n=80)	(10,0.007)	0.25	5.8	6.4	51.3	6.3	5.9	5.4	7.6	4.4
	(10,0.03)	0.5	8.1	9.7	61.2	10.3	6.7	7.1	6.5	6.2
	(10,0.06)	0.75	10.3	15.1	70.7	16.3	8.4	7.8	7.2	7.4
	(10,0.1)	1	14.9	24.7	81.8	28.4	12.5	10.0	9.8	8.4
	(10,0.15)	1.25	22.5	40.4	91.0	46.1	18.4	13.8	15.6	8.9
	(10,0.20)	1.5	31.1	57.4	96.5	64.0	25.2	17.1	21.2	9.6
	(10,0.25)	1.75	43.6	73.8	98.9	78.3	34.6	23.4	27.7	10.3
	(10,0.31)	2	56.1	86.8	99.7	89.7	46.2	30.8	36.4	12.7
	(10,0.36)	2.25	66.8	93.7	99.9	95.2	55.5	36.2	49.0	17.7
	(10,0.41)	2.5	76.3	97.1	100.0	97.6	66.4	44.1	57.9	22.5
	(10,0.46)	2.75	83.1	98.9	100.0	98.9	73.3	50.1	67.3	35.1
	(10,0.52)	3	89.8	99.6	100.0	99.6	81.7	58.0	76.5	40.4
Large Sample Size (n=150)	(10,0.007)	0.25	5.8	5.8	52.6	6.8	6.5	5.5	7.9	4.9
	(10,0.03)	0.5	8.6	10.6	63.7	12.8	7.5	7.2	8.4	6.2
	(10,0.06)	0.75	12.2	17.8	75.6	22.5	11.0	8.4	8.8	7.5
	(10,0.1)	1	20.7	33.1	88.1	43.1	17.1	12.6	11.8	8.6
	(10,0.15)	1.25	32.9	55.0	96.3	69.0	27.5	17.7	19.4	9.8
	(10,0.20)	1.5	48.3	75.6	99.2	87.2	39.4	24.4	35.2	11.7
	(10,0.25)	1.75	63.5	89.5	99.9	95.7	54.4	34.2	47.9	18.8
	(10,0.31)	2	78.9	97.5	100.0	99.3	69.3	44.3	63.0	27.0
	(10,0.36)	2.25	88.3	99.4	100.0	99.8	79.9	53.9	75.8	40.1
	(10,0.41)	2.5	94.1	99.9	100.0	100.0	87.7	62.6	84.1	50.7
	(10,0.46)	2.75	97.2	100.0	100.0	100.0	93.4	70.8	91.9	66.8
	(10,0.52)	3	99.0	100.0	100.0	100.0	97.0	79.3	96.1	77.1

At small sample size among classical tests of skewness the SK-2 performs best by attaining high power when the parameters of lognormal distribution is low. But, when the level of skewness is high then among classical tests SK-2 has maximum power and MED has the lowest power. Similarly, t-test is the better performer test when parameter values of lognormal distribution are high but at low parametric values both of the bootstrap tests have same power. At medium sample size (n=80) among classical tests, only SK-2 attains high power at low level of skewness. Similarly, bootstrap test has same power at low level of skewness. However, at high level of skewness among classical tests SK-2, SK-1 and

STDM tests have almost same power. In large sample size (n=150), all classical tests have same power except MED and SSS tests at high level of skewness. At this stage the power of MED test is very poor. While bootstrap test also shows high power.

It is also analyzed that SK-2 test with maximum power at each alternatives of skewness is obtained as best test while KS test is found as worst performing test among both categories of tests (see Figure 3) when sample size is small (n=40). Among classical tests of skewness SK-2 test performs best by attaining high power while MED test identifies as worst test when the sample size is small. Moreover, power of all tests of skewness is almost same except SK-2 when the level of skewness is 1 but then increases when the level of skewness becomes high. At medium sample size (n=80), power of SK-2 test is far better than other tests of skewness by attaining high power at each alternatives of skewness while KS test with less power is detected as worst test. SK-1 and STDM tests have almost same power at different alternatives of skewness. Moreover, MED test has worst performance by attaining low power at different level of skewness in the category of classical tests of skewness. Also, at medium sample size the performance of all tests of skewness get better than small sample size.

At large sample size (n=150), it is observed that performance of all tests of skewness at different alternatives of skewness is better than their performance at n=40 and n=80. Again, SK-2 test is still showing better performance by attaining high power at each alternatives of skewness when the sample size is large. Similarly, MED and KS tests have still classified as worst performer tests in category of classical and bootstrap tests, respectively, at all alternatives of skewness. However, KS test is the worst performer test among all tests by attaining low power at different alternatives of skewness. Hence, at large, medium and small sample sizes among all tests of skewness KS test shows poor performance while SK-2 test is the best performing test.

Power Computation in Mixture of Two Normal Distributions

The mixture of two normal distributions with different alternatives of skewness is taken to compare all skewness tests. It is observed that SK-2 test performs best as compared to all other skewness tests by attaining high power while KS test is found to be worst performer test (see Table 5). At small sample size (n=40), among classical tests of skewness SK-2 with high power is identified best test when the level of skewness is low. However, as the level of skewness is increases SK-2, SK-1 and STDM tests have almost same power while SSS and MED tests have the lowest power. At medium sample size (n=80) among classical tests, only SK-2 attains high power at low level of skewness. Moreover, at high level of skewness among classical tests SK-2, SK-1 and STDM tests attain same power pattern. Similarly, in the bootstrap tests category t-test has highest power while KS test shows worst performance by attaining low power.

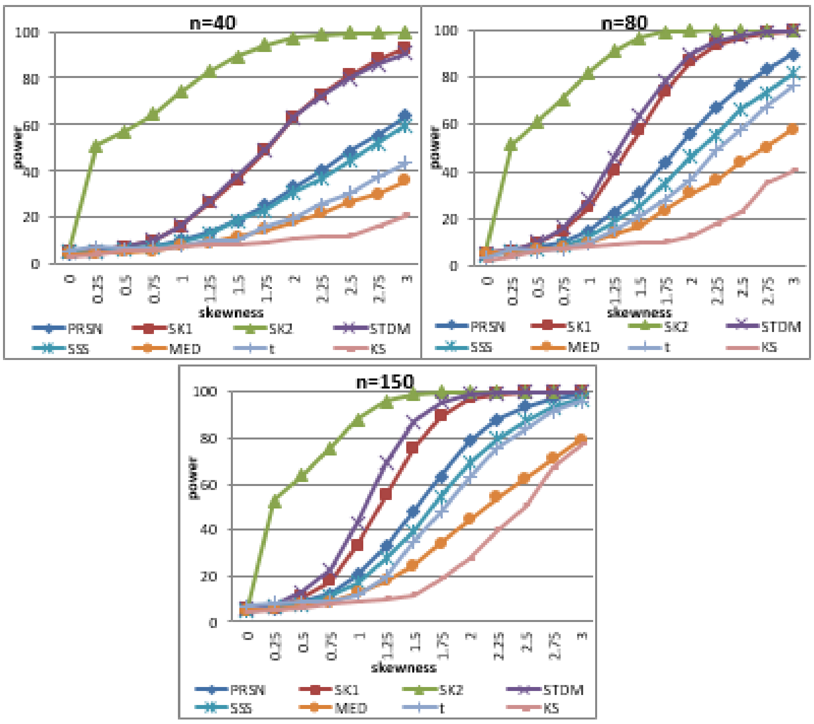

Fig. 3. Power Curve of Classical Tests and Bootstrap Tests in Lognormal Distribution

In large sample size (n=150) among classical tests at low level of skewness SK-2 test attains high power. But, as the level of skewness increases all other tests show better performance except MED, SSS and KS tests of skewness. At highest level of skewness among all tests of skewness PRSN, SK-2, SK-1, STDM and t tests have same power.

At small sample size (n=40), it is observed that SK-2 test performs best by attaining high power when the level of skewness increases (see Figure 4). Similarly, SK-1 and STDM tests also show better performance by attaining high power as the level of skewness increases. While, KS test shows poor performance by attaining low power at different alternatives of skewness.

Table 5
Power of Tests of Skewness for Mixture of Two Normal Distributions

	3rd Moment Theoretical Skewness	Classical Tests						Bootstrap Tests	
		PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Small Sample Size (n=40)	0	5.2	5.2	5.2	4.7	4.8	5.0	5.0	3.3
	0.25	5.6	8.7	44.6	9.8	5.9	5.2	10.1	5.9
	0.5	8.4	17.4	76.3	23.8	9.4	6.9	12.6	6.0
	0.75	12.6	31.2	89.5	44.6	15.8	18.3	17.1	7.0
	1	20.3	50.0	98.2	71.6	27.6	22.0	23.0	8.2
	1.25	28.3	69.1	99.8	89.8	34.7	33.9	27.8	8.4
	1.5	36.3	85.5	99.9	96.4	40.6	36.4	32.9	8.8
	1.75	46.3	94.8	100.0	98.5	43.6	40.7	38.0	9.0
	2	57.0	98.3	100.0	98.5	44.3	42.3	43.5	9.1
	2.25	68.1	98.5	100.0	98.7	44.4	43.7	47.4	9.4
	2.5	69.0	98.6	100.0	99.6	45.1	44.1	49.8	9.7
	2.75	69.5	98.9	100.0	100.0	46.2	44.8	53.7	10.0
	3	71.4	100.0	100.0	100.0	47.5	44.9	56.0	10.6
Medium Sample Size (n=80)	0.25	7.0	11.3	66.4	15.5	6.6	7.5	12.9	10.8
	0.5	14.2	25.7	87.7	42.7	13.8	15.2	21.1	12.8
	0.75	24.8	47.0	96.3	73.6	24.3	19.8	32.9	15.5
	1	43.4	67.8	99.9	95.6	37.8	26.5	47.4	20.8
	1.25	58.0	85.7	100.0	99.5	45.7	36.2	59.4	23.9
	1.5	70.7	96.3	100.0	99.9	46.6	40.7	69.2	25.5
	1.75	81.9	99.6	100.0	100.0	47.6	43.7	77.5	26.6
	2	89.7	100.0	100.0	100.0	48.0	46.0	83.4	26.6
	2.25	94.5	100.0	100.0	100.0	49.3	48.1	88.7	27.4
	2.5	95.5	100.0	100.0	100.0	50.1	48.7	89.4	28.9
	2.75	96.5	100.0	100.0	100.0	52.3	50.4	91.3	29.8
	3	97.1	100.0	100.0	100.0	53.3	50.5	94.3	32.0
Large Sample Size (n=150)	0.25	10.8	15.3	86.8	25.1	9.7	8.8	17.5	7.0
	0.5	25.4	36.6	98.9	68.7	22.6	12.4	35.1	10.2
	0.75	48.2	62.7	99.8	84.8	39.3	18.5	54.4	14.6
	1	73.6	83.1	100.0	96.9	55.9	27.0	76.2	21.7
	1.25	86.5	94.5	100.0	99.9	60.2	33.7	87.7	27.6
	1.5	94.6	99.1	100.0	100.0	61.5	39.9	93.7	33.3
	1.75	98.1	99.9	100.0	100.0	62.2	45.7	97.6	38.7
	2	99.4	100.0	100.0	100.0	64.0	48.8	98.8	42.2
	2.25	99.9	100.0	100.0	100.0	65.8	52.9	99.8	47.6
	2.5	100.0	100.0	100.0	100.0	66.7	56.4	100.0	52.2
	2.75	100.0	100.0	100.0	100.0	67.8	58.9	100.0	53.8
	3	100.0	100.0	100.0	100.0	69.0	63.0	100.0	56.8

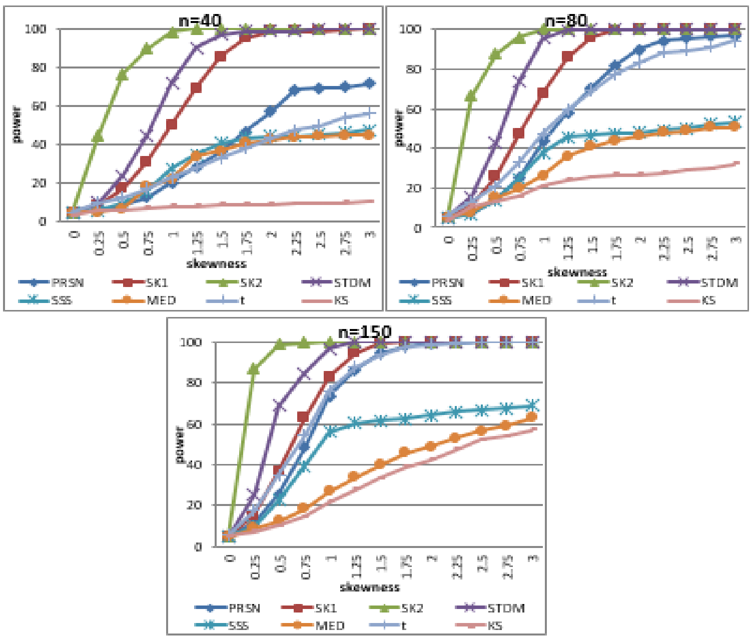

Fig. 4. Power Curve of Classical and Bootstrap Tests in Mixture of Two Normal Distribution

When the size of sample increases to 80 then SK-2 test performs best by attaining high power while KS test has kept its previous position (worst performer). The performance of MED and SSS tests are very poor by attaining low power at different alternatives of skewness and identified as worst performing tests in the category of classical tests.

At large sample size (n=150), a similar pattern of almost all tests have been analyzed as shown for small and medium sample sizes. Here, PRSN and t-test have almost the same power while power of SSS test gets better when the level of skewness becomes high. MED and KS tests show poor performance by attaining low power at different alternatives of skewness. Hence, among all tests of skewness MED and KS tests show poor power for all sample sizes while only SK-2 shows best power in all sample sizes.

Power Computation in Mixture of Two Uniforms and Normal Distribution

When data are generated from mixture of two uniforms and normal distributions (i.e. DGP-6) then power of all classical and bootstrap tests of skewness are compared and very different results are obtained as compared to results of DGP-2, DGP-3, DGP-4 and DGP-5 (see Table 6). It is observed that all the classical tests of skewness detect zero skewness while the distribution is skewed. Also, we have observed that power of some of the classical test of skewness is equal to the size of the test. Moreover, SK-2 test have showed excellent performance in DGP-2, DGP-3, DGP-4 and DGP-5 but fails to detect high power for all sample sizes under DGP-6. While, bootstrap tests of skewness detect high power for all sample sizes under DGP-6. Hence, all classical tests of skewness completely fails in detecting skewness even though distribution is skewed while bootstrap tests of skewness shows high power in detecting skewness.

4. EMPIRICAL EXAMPLES

Most of classical tests of skewness based on the third central moment, there are situations where the third central moment is zero but there is skewness.

Table 6

Power of Tests of Skewness for Mixture of Two Uniforms and Normal Distributions

	Classical Tests						Bootstrap Tests	
	PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Small Sample Size (n=40)	0	0	0	0	0	0	96.2	88.9
	5.5	2.6	0	2.6	4.4	2.5	76.7	73.5
	0	0	0	0	0	0	99.3	90.7
	4.7	1.6	0	1.4	4.4	4.4	83.5	79
	1.1	3.8	0	1.9	3.7	2.4	89.6	82.4
	0	0	0	0	0	1	85	81
Medium Sample Size (n=80)	0	0	0	0	0	0	99.9	99.9
	6.5	4.7	0	3.4	5.7	2.6	82	83
	0	0	0	0	0	0.	100	99.7
	5.6	0.3	0	0.3	3.4	5.8	84.3	81
	1.7	4.1	0	2.9	4.9	4.6	91.9	85
	0	0	0	0	0	0	88	85.5
Large Sample Size (n=150)	0	0	0	0	0	0	100	100
	6.4	4.7	0	3.2	5.5	3.5	99.5	95.5
	0	0	0	0	0	0	100	100
	6.2	0	0	0	4.4	6.7	88.7	90
	2.5	4.9	0	1.9	4.7	5.5	95.5	89.2
	0	0	0	0	0	0	91.3	88.6

Table 7

Results of Classical and Bootstrap Tests of Whole Sale Price Index of Netherland

	Classical Tests						Bootstrap Tests	
	SRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
Calculated Skewness	-0.78	0.93	0.98	-0.03	-0.24	-0.36	3.54	0.44
Critical Value	0.48	1.66	11.22	0.52	0.6	0.23	(-2.09,2.09)	0.4
Conclusions	Insig;	Insig;	Insig;	Insig;	Insig;	Insig;	Sig;	Sig;

A real world example of data of Whole Sale Price Index (WSPI) of Netherland having skewed distribution is elaborated (see Figure 5), whose third moment is zero even there exists skewness which is only detected by bootstrap tests while classical tests of skewness are unable to detect skewness (see Table 7). These results indicate the importance of bootstrap tests whenever classical tests of skewness are unable to detect skewness.

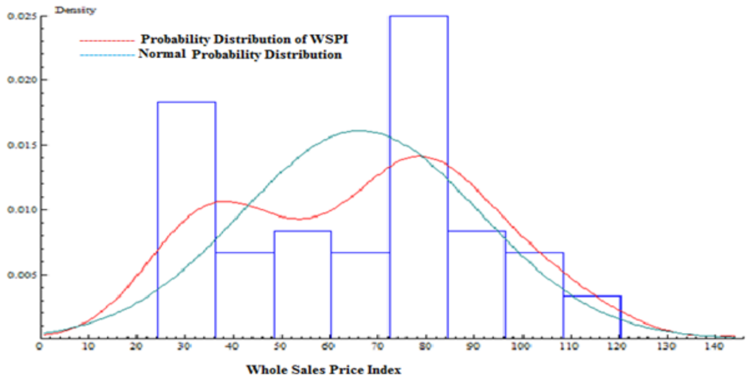

Fig. 5. Distribution of Data of Whole Sales Price Index (WSPI) of Netherland

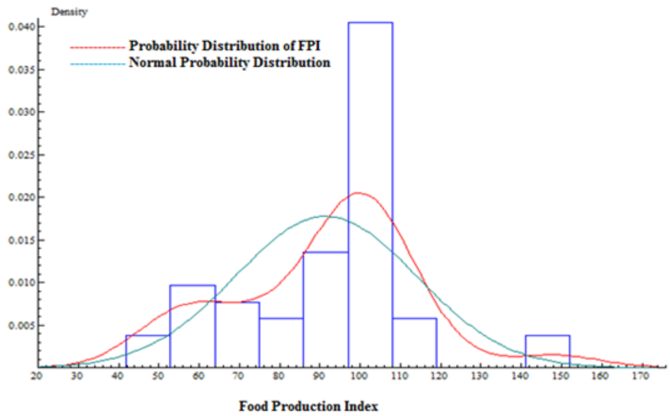

Fig. 6. Distribution of Food Production Index of Antigua and Barbados

There is another real world example of data of food production index of Antigua and Barbados having skewed distribution (see Figure 6), whose third central moment is zero in the presence of skewness but classical tests are fail to detect it while bootstrap tests have identified this distribution as skewed (see Table 8). There are also many more other real world examples of different countries data, the distribution of which is detected as skewed through histogram but classical tests of skewness fail to identify them skewed while bootstrap tests do. Similarly a data analysis of Pakistan by Raza et al. (2020), Latin America and Caribbean countries data analysis by Raza et al. (2023) and BRICS countries data analysis by Raza et al. (2024) can be test to get similar results

Hence, it is observed from the data of whole sale price index of Netherland and food production index of Antigua and Barbados that all classical tests of skewness are unable to detect skewness while bootstrap tests have successfully spotted skewness in the data. An attempt can be made for the Pak-Chinese Rupees-Yuan comparison and the analysis for this has been done by Waheed et al. (2023).

Table 8

Results of Classical and Bootstrap Tests of Skewness of Food Production Index of Antigua and Barbados

	Classical Tests						Bootstrap Tests	
Calculated Skewness	PRSN	SK-1	SK-2	STDM	SSS	MED	t	KS
	- 0.88	1.08	1.02	0.02	-1.45	-0.55	2.93	0.47
Critical Value	0.49	1.65	9.19	0.52	0.58	0.24	(-2.21,2.21)	0.39
Conclusions	Insig;	Insig;	Insig;	Insig;	Insig;	Insig;	Sig;	Sig;

5. CONCLUSION & RECOMMENDATION

This study concludes that SK-2 test and MED test are identified as best and worst performing tests, respectively, in all sample sizes in the category of classical tests of skewness. While, in the category of bootstrap tests t-test outperforms KS test by attaining high power at all alternatives whether sample size is small, medium or large. Overall, it is observed that SK-2 test with maximum power at all alternatives corresponding to small, medium and large sample sizes is detected as most powerful test among all classical and bootstrap tests of skewness while KS test is considered as worst performer test when DGP-2, DGP-3, DGP-4 and DGP-5 are considered. However, there are certain situations where the third moment is zero despite having skewed distribution. In these cases the classical tests did not perform well, e.g. for the mixture of two Uniforms and Normal distribution (under DGP-6) which is skewed distribution in which the power of classical tests remains in between 0 and 7%. But, the power of bootstrap test remains between 74% and 100% and outperform classical tests. Real data examples are also furnished where third moment is zero but the distribution is not symmetric. In these examples, classical tests of skewness were unable to detect skewness while bootstrap tests have successfully detect it.

Therefore this study recommends classical tests of skewness are more appropriate to apply if third moment of data is non-zero. However, if the third moment of data is zero and the empirical histogram shows significant deviation from symmetry then one should apply bootstrap tests for skewness.

Conflict interests

The authors has declared that no competing interests exist.

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